International Journal of Theoretical Physics, Vol. 2, No. 2 (1969), pp. 157–160.

Qualitative Features of Quantized Gravitation†

ARTHUR KOMAR

Belfer Graduate School of Science
Yeshiva University
New York, New York 10033

Abstract

From a re-examination of a discussion by Bohr and Einstein of a gedanken experiment designed to violate the uncertainty relations, it is suggested that a qualitatively new feature of the merging quantum theory of gravitation is that it may provide a new understanding of the 'reduction of the wave packet' of quantum mechanics. In brief, for finitely massive observers, half of the classical dynamical variables must be employed to specify the frame of reference, whereas only the remaining half of the dynamical variables are available for unequivocal observation. Observers whose frames of reference cannot be related by definite C-number transformations would, in general, 'reduce wave packets' differently.

1. *Introduction*

Despite the considerable degree of activity being currently expended in the development of a quantum theory of gravitation, virtually everyone is in agreement that the gravitational modification to scattering cross-sections and nuclear energy levels is so minute as to be immeasurable. Were this the only expected effect of the theory one might well question the value of this effort. The purpose of this note is to indicate a qualitatively new feature which the quantized version of general relativity may provide in the clarification of our understanding of quantum processes.

2. *The Bohr-Einstein Gedanken Experiment*

In a famous paper (Bohr, 1961) Bohr discussed an attempt by Einstein to violate the uncertainty relation $\Delta E \Delta t > h$. In summary Einstein's argument ran as follows. A box is equipped with a source of radiant energy, a shutter, and a clock preset to open the shutter for a precise time interval Δt. Prior to the opening of the shutter one has an arbitrarily long time to weigh the box (including its contents) thereby determining its energy to arbitrary accuracy. After the shutter has opened, permitting some radiation to escape, one again has an arbitrarily long time available to redetermine,

---

† Supported in part by the United States Air Force Office of Scientific Research under Grant AFOSR 68-1524.

by means of weighing, the energy of the box and its contents. The difference between the two determinations of the energy, which has evidently been specified to arbitrary accuracy, is equal to energy radiated in the predetermined time interval Δt that the shutter was open. It would therefore appear that, for the wave train emitted, the uncertainty relation $\Delta E \Delta t > h$ can be violated.

In order to exhibit this violation of the uncertainty principle Einstein had to employ the principle of equivalence of gravitational mass and inertial mass, as well as the relativistic relationship of the equality of inertial mass and energy. Bohr observed that in order to analyze properly the above gedanken experiment it was essential to use a consistent theory. Newtonian gravitational theory is therefore not appropriate for the analysis since it is neither consistent with the principle of equivalence nor with relativity theory. However, by employing a theory of gravitation which is consistent with the essential properties used in the analysis of the experiment, namely the general theory of relativity, Bohr demonstrated that, in the course of the weighing of the box, an additional uncertainty in the time is introduced, of precisely the correct amount to restore the validity of the uncertainty relation. The source of the additional uncertainty in the knowledge of the preset time interval Δt is a consequence of the fact that, in the course of the weighing, the clock is suspended at an uncertain location in a gravitational field, while an essential feature of Einstein's theory of gravitation is that clocks run at different rates in differing gravitational fields. (The reader is recommended to study the original paper (Bohr, 1961), since the argument is rather intricate and subtle.)

3. *Relative Frames*

Although Bohr's analysis of the gedanken experiment fully demonstrated the impossibility of violating the uncertainty principle by means of weighings, it is of interest to pursue the problem a little further. It is clear that the preset time interval Δt of the clock in the box is an interval of proper time of (all observers co-moving with) that clock. What has become uncertain in the course of weighing is not that interval of proper time, but rather the relationship between that proper time and the coordinate time in the laboratory where the weighing was performed, and consequently, where energy of the wave train was determined. Were it possible to regard observers within the box equally legitimate with those of the laboratory we see that different observers could reduce the same wave packet in radically different (in fact, complementary) ways: the observer within the box finds the time interval of the wave packet arbitrarily well defined, while the observer in the laboratory finds the energy of the packet arbitrarily well defined. From this point of view, the uncertainty would appear to reside in the relationship between the frames of reference of the different observers.[†]

Of course, in the space-time of classical physics the observer in the laboratory and the observer in the box are not equally legitimate. For, if the laboratory observer is in an inertial frame in the course of weighing the box, the observer in the box is most certainly not in an inertial frame. The latter's fluctuating position while being weighed will be experienced as a fluctuating gravitational field whose presence cannot be ignored. Even should the laboratory frame not be regarded as inertial in view of the uniform gravitational field assumed to be available there, to the extent that the laboratory is customarily regarded as infinitely massive, no fluctuation would be experienced by such a frame. Although in classical general relativity one must exclude infinitely massive objects, one still is able to distinguish inertial motion from fluctuating motion in an invariant way. Thus the possibility remains for a preferred set of laboratory observers whose interpretation of experiments may be regarded as having a greater degree of legitimacy.

When we turn to the consideration of the quantum theory of gravitation we find that the Riemannian manifold itself has dissolved. The Cauchy data which is required to determine the classical space-time may be divided into canonically conjugate pairs (Dirac, 1951) only half of which remains available in the quantum theory. The canonically conjugate pairs may be understood as the observables which characterize the spatial geometries which may be imbedded in the same Ricci-flat four-geometry, and the observables which characterize the structure of the inequivalent stackings of spatial geometries into a four-geometry (Komar, 1968). In view of the fact that in the quantum theory both kinds of observables cannot be specified simultaneously to arbitrary accuracy, it is no longer possible to have a classical Riemannian manifold containing a preferred set of inertial or non-fluctuating observers.

4. *Conclusion*

If we return to the Einstein gedanken experiment, we find that in the context of the quantum theory of gravitation both the observer in the box and the observer in the laboratory must be finitely massive and on equal footing. Each observer must regard his frame of reference as classical, and describe the other observer by means of a wave function. It is evident that an ordinary space-time coordinate transformation cannot connect the frames of reference of the two observers, there being no classical space-time

---

[†] In a private communication, Y. Aharonov has pointed out that an analogous argument can be given for understanding the $\Delta x \Delta p > h$ uncertainty relation also as a statement about the relationship between frames of reference of observers who differ in their interpretation of which of the complementary pair of variables x, p is diagonal.

Dirac, P. A. M. (1951). *Physical Review*, **83**, 1018.
Komar, A. (1968). Hamilton-Jacobi Version of General Relativity. *Physical Review*. (To be published.)

on which existence they can agree. It is conceivable that their frames of reference can be related by means of an operator coordinate transformation, that is, by means of a coordinate transformation which involves explicit reference to the metric tensor, as would be the case if the specification of frames of reference required coordinate conditions. However, the quantum theory of gravitation has as yet not been developed to the point where one can give a definite answer on this point. One thing, though, is quite clear: the two observers of the gedanken experiment will not agree on how the wave packet of the emitted radiation will be reduced—and each description will be legitimate! The uncertainty principle will be transferred from the wave packet to the relationship between the frames of reference of the two observers, and ultimately to the structure of the space-time itself. What we gain in the process is the qualitative understanding of why it is at the discretion of the observer to arrange his apparatus in order to measure either of a pair of canonical variables at the expense of loosing information about the other. Essentially, the arrangement of the apparatus of the experiment determines whether the frame of reference which is being established is being anchored relative to the spatial geometries or relative to the stacking of the spatial geometries (or perhaps some mixture of the two).

A more detailed analysis must, of course, await the development of a quantum theory of gravitation. The intention of this note is to encourage one to expect more from such a theory than merely an unmeasurable correction to some scattering cross-section.

References

Bohr, N. (1961). *Atomic Physics and Human Knowledge*, pp. 53–55. Science Editions, Inc., New York.